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Abstract

The study aims to verify the relationship between the unemployment rate and economic growth in European Union (EU) regions. As the most important macroeconomic relationship, the significance of the dependence between the labour market situation and the output growth is widely known and considered. Analysis in this research was conducted using data for 229 EU regions on the NUTS-2 level in the years 2013–2019. In order to verify the relationship between the unemployment rate and the output growth, the spatio-temporal models for pooled time series and cross-sectional data (TSCS) were estimated. The Fitted Trend and Elasticity Method of verifying Okun’s law was used in the analysis, wherein the deterministic trend factor was enriched with the spatial element. Educational attainment as the additional explanatory variable was included in the models. The neighbourhood between regions was quantified based on two criteria: (1) common border criterion – related to the possibility of population migrations, and (2) similarity of the unemployment rate criterion – related to the imitation effect in the issue of introduced rules and regulations on the labour market by regional governments. One of the hypotheses verified in the investigation is the superiority of the economic neighbourhood over the geographical neighbourhood.

Keywords: economic growth, European Union, Okun’s law, spatio-temporal models, unemployment rate

Introduction

The problem of unemployment is one of the essential issues in macroeconomic analyses. This is widely known that the persistent regional unemployment disparities in the European Union occur (Patuelli, R. et al. 2012; Halleck Vega, S. and Elhorst, J.P. 2016) Increasing the number of unemployed persons is a problem in both developing and developed regions. A lot of determinants significantly influence the regional unemployment disparities. For example, regional demand and supply factors, the labour migration (Andrews, R. 2015; Lados, G. and Hegedüs, G. 2016) and amenities are the most important factors. Positive changes in these factors can disincentive to migration, compensating for relatively high unemployment rates (Rios, V. 2017). Moreover, the institutional decisions providing restrictions or incentives influence the individual decisions regarding labour demand, supply and wages paid, which changes the level of the unemployment rate (Boeri, T. 2011). Capital inadequacy can influence the decrease in the employed people above all in the developing countries. In turn, technological progress is the primary reason for the increased unemployment rate in developed regions (Soylu, Ö.B. et al. 2018). Moreover, the national labour market regulation and labour market institutional system play significant role in the creation of unemployment rate in every economy. Whereas, as the most important determinant of unemployment, economic growth is pointed out. Arthur M. Okun proved the negative relationship between

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the unemployment rate and Gross National Product (GNP) based on the data for the United States (US) in the years 1947–1960 (Okun, A.M. 1963). He concluded that each 1 percent increase in GNP led to a 0.3 percent decrease in the unemployment rate.

Since that time, the relationship between the unemployment rate and economic growth has been widely concerned in macroeconomic analyses. This negative short-run dependence is known as Okun’s law. Previous studies conclude that the relationship formulated by Okun is stable in many countries (Ball, L. et al. 2017) and possible instability is visible in terms of the economic slowdown (Cazes, S. et al. 2011). In turn, in some studies, authors concluded that the discrepancy in Okun’s relationship between regions within one country occurs (Adanu, K. 2005; Binet, M. and Facchini, F. 2013; Durech, R. et al. 2014).

There are three methods of verifying Okun’s relationship: (1) Trial Gaps Method, (2) First Differences Method, and (3) Fitted Trend and Elasticity Method (Barreto, H. and Howland, F. 1993). The first two methods treated analysed processes as the processes stationary in the variance. Instead, within the meaning of the third method, processes are stationary in the average. The First Differences Method is characterized by the greatest popularity in previous studies. In this research, the Fitted Trend and Elasticity method is used, but the deterministic trend factor is enriched with the spatial element (originally, only time tendency was considered).

In this study, the relationship between the unemployment rate and its main determinants – economic growth and educational attainment – is analysed. The study’s main aim is to show that the economic growth and unemployment rate are significantly related in European Union regions. Moreover, the stronger importance of economic similarity between territorial units than their near geographical location in the considered relationship is verified. As a space and time range of the research, the NUTS-2 European Union regions in the years 2013–2019 were chosen (due to lack of data for the unemployment rate, Croatian regions were omitted). In the verification of the mentioned relationship, the spatial and spatio-temporal dependencies were included. Many researchers pointed out the importance of spatial connections in the unemployment rate analyses (Overman, H. et al. 2002; Patachinni, E. and Zenou, Y. 2007; Halleck Vega, S. and Elhorst, J.P. 2016). In this study, two types of neighbourhood connections were considered. The first is the geographical neighbourhood (associated with the possibility of migration), and the second is the economic neighbourhood (associated with the unemployment rate similarity). Two research hypotheses were verified in this investigation: (1) Output growth and education have a significant positive impact on the labour market conditions in the EU regions, and (2) Economic similarity of regions is more important than a geographical neighbourhood in the formation of Okun’s relationship.

There are many studies considering Okun’s relationship at the regional level, and different methods are used in order to verify it. A couple of studies pertain to Okun’s relationship in the Spanish provinces (Villaverde, J. and Mazá, Á. 2007, 2009; Clar-Lopez, M. et al. 2014; Cháfer, C.M. 2015; Bande, R. and Martín-Román, Á. 2018; Guisinger, A.Y. et al. 2018; Cutanda, A. 2023). All of these investigations are based on non-spatial analysis, and only Villaverde, J. and Mazá, A. (2007) considered the Fitted Trend and Elasticity Method with the quadratic trend. Bande, R. and Martín-Román, Á. (2018) estimated a simple model for the first differences of processes and also for trial gaps. The same method of research took Clar-Lopez, M. et al. (2014). The subject of the investigations in terms of the relationship between unemployment rates were also regions from other European countries, e.g., Italian provinces (Salvatí, L. 2015), Finnish regions (Kangasharju, A. et al. 2012), Greek regions (Apergis, N. and Rezitis, A. 2003), and also Czech and Slovak regions (Durech, R. et al. 2014). Most of these studies underline the regional disparities in the unemploy-
ment rate formation rely on their economic development. In turn, Yerdelen, F. and İcen, H. verified Okun’s relationship for NUTS-2 level regions from 20 European countries using panel data models (Yerdelen, F. and İcen, H. 2019). Apart from the studies for European countries, it is possible to find analyses of the mentioned relationship at the regional level in the United States (Huang, H.C. and Yeh, C.C. 2013; Guisinger, A.Y. et al. 2018), Canada (Adanu, K. 2005), Indonesia (Sasongko, G. et al. 2020), and South Africa (Kavese, K. and Phiri, A. 2020).

The spatial factor in the analysis of the dependence between the unemployment rate and output growth is also included in previous studies. Duran, H.E. considered this dependence using spatial panel data models using data for 26 Turkish NUTS-2 regions (Duran, H.E. 2022). Spatial regression models were used in the analyses concerning the unemployment rate in the United States (Montero-Kuscevic, C.M. 2011; Perreira, R.M. 2013), and EU15 NUTS-2 regions (Herwartz, H. and Niebuhr, A. 2011). In turn, Adolfo Maza conducted the analysis for the widest space range in the mentioned relationship (Maza, A. 2022). He considered the unemployment rate in 265 European regions between 2000 and 2019.

Methodology

In this research, spatial econometric methods were used in order to verify the Okun’s relationship. Spatial econometric models contain the influence of process changes in the neighbouring regions on the same process in the established region. In the regional analyses, connections between nearby and also similar (in the economic context) units are very important, in particular in case of the unemployment analysis. The economic conditions and possibilities of the neighbours can encourage people to jobs migrations, changing the labour situation in the considered unit.

In the first part of the investigation, the spatio-temporal structure of processes was analysed. The structure is composed of the spatio-temporal trend and spatio-temporal autocorrelation. Initially, spatio-temporal trend models were considered, which general form is as follows (Cressie, N.A.C. 1993):

\[ p(s_i, t) = \sum_{k=0}^{p} \sum_{m=0}^{p} \sum_{l=0}^{p} \theta_{km} x_i^k y_i^m t^l, \]

where \( s_i = [x_i, y_i] \) denotes unit’s location coordinates on the plane (longitude and latitude, respectively), \( i = 1, 2, ..., N \) are indexes of spatial units, and \( p \) means the polynomial trend degree \( (k + m + l \leq p) \) but \( t \) indicates time.

Simultaneously, the spatio-temporal autocorrelation presence as the second element of the spatio-temporal structure was checked. The spatial autocorrelation is tested using Moran statistics, which takes the following form (Moran, P.A.P. 1948; Schabenberger, O. and Gotway, C.A. 2005):

\[ i = \frac{1}{\sum_{t=1}^{n} \sum_{t=1}^{n} w_{tt} \left( y_{tt} - \bar{y} \right) \left( y_{tt} - \bar{y} \right)^t} \sum_{t=1}^{n} \bar{y}^t w^t \sum_{t=1}^{n} \bar{y}^t w^t \]

where \( y_{tt} \) is the observation of the process in the \( i^{th} \) region in time \( t \), \( \bar{y} \) denotes the average value of the process, \( W^t \) is the block matrix of spatio-temporal connections between units given as Szulc, E. and Jankiewicz, M. (2018):

\[ W^t = [w_{ij}]_{N \times N} \]

wherein \( W_1 = W_2 = ... = W_T \) are standard spatial connectivity matrices quantified for a certain year. In this study, these matrices are the same for all years.

In this research, two types of row-standardized to unity matrices were adopted. The first of them is based on the common border criterion (marked as \( W \)). Therefore, two regions are neighbours if they have a common land border. In turn, the second defines the neighbourhood as the economic similarity \( (D) \) — regions are neighbours if the difference between their unemployment rate level in the last year of the investigation does not exceed a certain specific value (established as 0.8% — the 15th percentile of differences between the unemployment rate in all regions). The procedure of building the economic dis-
tance matrix is presented by Jankiewicz, M. and Szulc, E. in their study (Jankiewicz, M. and Szulc, E. 2021).

Statistically significant Moran’s I coefficient signalizes the presence of spatial autocorrelation. Its positive value denotes that territorial units create clusters of the regions with a similar level of the analysed phenomenon. In turn, the negative sign of statistics points out that neighbouring regions are characterized by different values of the considered process. Non-significant statistics testifies to a random distribution of the process values in space.

Next, the spatio-temporal models of the relationship between the unemployment rate, economic growth, and educational attainment were considered. The general form of the TSCS model (pooled time series and cross-sectional data model) is as follows:

\[ Y_{i,t} = \beta_1 X_{1,i,t} + \beta_2 X_{2,i,t} + \varepsilon_{i,t}, \quad (4) \]

where \( Y_{i,t} \) denotes the unemployment rate in the \( i \)th region in time \( t \), \( X_{1,i,t} \) and \( X_{2,i,t} \) are levels of Gross Domestic Product per capita and educational attainment, respectively (all expressed in natural logarithms). In turn, \( \varepsilon_{i,t} \) indicates the spatio-temporal random component, but \( \beta_1 \) and \( \beta_2 \) are the structural parameters. Logarithms of variables cause that parameter \( \beta_1 \) is the elasticity parameter in the Fitted Trend and Elasticity Method in the Okun’s law verification. Model (4) is deprived of constant due to all considered variables are filtered out from deterministic spatio-temporal trend, which is responsible for their average values.

In terms of global spatial autocorrelation in the residuals of the model (4) the character of the spatial dependence was determined using Lagrange Multiplier (LM) tests in the basic and robust version (Anselin, L. et al. 2004). Including spatially lagged explanatory variables in the models, the spatio-temporal Durbin model (STDM) and spatio-temporal hybrid model (STHM) are given as follows:

\[ Y_{i,t} = \beta_1 X_{1,i,t} + \beta_2 X_{2,i,t} + \sum_{j \neq t} w_{ij,t} X_{1,j,t} + \sum_{j \neq t} w_{ij,t} X_{2,j,t} + \varepsilon_{i,t}, \quad (5) \]

\[ \eta_{i,t} = \lambda \sum_{j \neq t} w_{ij,t} \eta_{j,t} + \varepsilon_{i,t}, \quad (6) \]

where \( Y_{i,t}, X_{1,i,t}, X_{2,i,t}, \varepsilon_{i,t}, \beta_1, \beta_2 \) – as above, \( \sum_{j \neq t} w_{ij,t} \) – spatially lagged variables, \( \sum_{j \neq t} w_{ij,t} \eta_{j,t} \) – spatially lagged random process, \( \theta_1, \theta_2, \rho, \lambda \) – structural parameters. Parameters \( \rho \) and \( \lambda \) evidence the spatial dependence between neighbouring territorial units.

## Spatial and spatio-temporal structure of processes

Data used in this study concern the unemployment rate (marked as \( Y \)), Gross Domestic Product per capita (\( X_1 \)), and educational attainment level, understood as the percent of the population that graduated upper secondary and post-secondary (not tertiary) school (\( X_2 \)) in the European Union regions in the years 2013–2019. Better economic conditions in regions favor the creation of new workplaces, whereas the higher education level of society improves chances of getting a job. The indicators used in the research are only a few of many that significantly impact regional unemployment, but they are considered the most important. Apart from them, e.g., the innovation level and the economic structure of regions are important. The first is not included due to data unavailability for the whole considered area, while the second will be of interest for further research. Moreover, in case of unemployment, the age structure of the population and distance from main urban centres play an important role. The established period is the maximum period that can be analysed in light of the data availability. Moreover, due to a lack of data characterizing the unemployment level, Croatia’s regions were omitted. All data come directly from the European Statistical Office (EUROSTAT) database – https://ec.europa.eu/eurostat/data/database (accessed: 04.07.2022).

In the first part of the research, spatial distributions of considered processes were presented. These distributions were shown in three figures (for variable \( Y \), for variable \( X_1 \) and for...
variable $X_j$ in the extreme years of the analysis. In each figure, part (a) indicates the distribution in 2013, but part (b) refers to spatial differentiation in 2019. EU regions were divided into four groups using positional measures of the descriptive statistics (median and quarter deviation).

As we can see in Figure 1, the highest unemployment rate in 2013 was observed in the Iberian Peninsula regions, Greek regions, and the units located in Southern Italy. Moreover, relatively high unemployment was noted in most of Eastern Europe NUTS-2 level regions (above all in the regions located in Lithuania, Poland, and the Slovak Republic). On the other hand, Austrian and West German regions were characterized by the best labour market conditions – the unemployment rate was relatively low. This is worth noting the low level of the considered variable in the North Romanian regions. In 2019 the situation in the labour market in the EU was slightly different than in 2013. The unemployment level in all Francian units was above the median in the last year of the investigation. Instead, the

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Fig. 1. Spatial distribution of the unemployment rate in EU regions in the years 2013 (a) and 2019 (b)
variable values in 2013 were more diversified between these regions. The relative deterioration of the situation concerns North Italian provinces and Scandinavian regions as well. In contrast, most of the Polish provinces found themselves in units with unemployment rate values below the median, which denotes the relative improvement in this part of the European Community.

Based on spatial distributions of the unemployment level in the EU regions, it can be presumed that values of the considered variable exhibit a certain tendency in space. Therefore, the spatial factor in the analysis should be included. In this connection, the spatio-temporal trend models in the following part of the study were concerned.

Seeing the spatial distributions of GDP per capita (Figure 2), we can note that the spatial differentiation of process values in both analysed years was analogous. Relatively high economic growth (with the values of GDP per capita above median) was observed in the Central EU regions (located in Austria,

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**Fig. 2.** Spatial distribution of the GDP per capita (PPS) in EU regions in the years 2013 (a) and 2019 (b)
West Germany, North Italy, and Benelux countries). Also, a high GDP level was noted in the Scandinavian and South Ireland regions. On the other hand, the relatively less developed units were located above all in the eastern part of the European Community, except for regions Bratislava (SK01), Praha (CZ01), Warszawski stołeczny (PL91), Sostinės (LT01), and București-Ilfov (RO32). Three mentioned units belonged to the group of regions with the highest values of the GDP per capita in 2019. A similar situation was observed in the Iberian Peninsula provinces. Almost all regions were classified into the groups of very low and low economic growth levels. Only two regions were characterized by economic growth above the median – one Spanish and one Portuguese (Madrid – ES30 and Área Metropolitana de Lisboa – PT17, respectively). As in the case of the unemployment rate, certain spatial tendencies in the formation of GDP per capita values were observed.

Figure 3 presents spatial distributions of educational attainment.

Fig. 3. Spatial distribution of the educational attainment level in EU regions in the years 2013 (a) and 2019 (b)
The percent of graduates in the upper secondary and post-secondary school (excluding tertiary education) is considered. It is worth seeing the possibility of the division of EU regions into two parts in both extreme years of the study (more visible in 2019). The first was in the eastern part of the European Community, where almost all regions were characterized by a relatively high percentage of graduates in upper secondary and post-secondary school. Instead, the western part of the mentioned area was dominated by units with the values of the considered process below the median. It is worth noting the lowest level of the variable $X_1$ in the French and Spanish regions, where the unemployment rate was relatively high. Units with the low and high values of the educational attainment process created two almost coherent areas, which lead to presumption about a certain spatial tendency in their formation.

The observations made based on the spatial distributions of all processes allowed us to consider the two-dimensional deterministic trend (with the spatial and time factors) in order to filter out long-term tendencies. Table 1 shows the results of estimation and verification of the spatio-temporal trend models for all variables. In the models, only the statistically significant parameters were left.

It is a difference in the degree of trend obtained for the variable $Y$ and the two remaining variables. The unemployment rate in the period 2013–2019 was shaped according to the second-degree spatio-temporal trend. Considering estimates of parameters $\theta_{100}$ and $\theta_{010}$ for variables $X_1$ and $X_2$ we can conclude that their values averagely have been growing in the western-northern and eastern-northern directions, respectively. This confirms the insights visible in figures 2 and 3. Moreover, positive estimates of parameter $\theta_{001}$ indicate the average increase of the GDP per capita and educational attainment in the years 2013–2019. In addition, an average decrease of the variable $Y$ towards the north and east should be noted (negative estimations of parameters $\theta_{010}$ and $\theta_{100}$, respectively).

The low level of the determination coefficient $R^2$ is the characteristic feature of the spatial and spatio-temporal trend models. However, we can see that the least coherent values formation in space and time was pertaining to Gross Domestic Product per capita.

Additionally, the Moran test evaluating the dependence between neighbouring regions was conducted. In the spatial autocorrelation analysis, two types of neighbourhood matrices were used. First of them defines neighbouring regions as regions with a common land border (geographical neighbourhood – $W$). Instead, the second points out that two units are neighbours if they had a similar level of the unemployment rate in 2019 (eco-

**Table 1. The results of estimation and verification of the spatio-temporal trend models**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unemployment ($Y$)</th>
<th>GDP per capita ($X_1$)</th>
<th>Educational attainment ($X_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>p-value</td>
<td>Estimate</td>
</tr>
<tr>
<td>$\theta_{000}$</td>
<td>15.6290</td>
<td>0.0000</td>
<td>8.9967</td>
</tr>
<tr>
<td>$\theta_{100}$</td>
<td>-0.0185</td>
<td>0.0000</td>
<td>-0.0190</td>
</tr>
<tr>
<td>$\theta_{010}$</td>
<td>-0.5110</td>
<td>0.0000</td>
<td>0.0259</td>
</tr>
<tr>
<td>$\theta_{001}$</td>
<td>–</td>
<td>–</td>
<td>0.0301</td>
</tr>
<tr>
<td>$\theta_{200}$</td>
<td>0.0008</td>
<td>0.0000</td>
<td>–</td>
</tr>
<tr>
<td>$\theta_{101}$</td>
<td>0.0048</td>
<td>0.0000</td>
<td>–</td>
</tr>
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<td>$\theta_{011}$</td>
<td>-0.0014</td>
<td>0.0123</td>
<td>–</td>
</tr>
<tr>
<td>$\theta_{002}$</td>
<td>-0.0104</td>
<td>0.0000</td>
<td>–</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.5432</td>
<td>0.3315</td>
<td>0.5318</td>
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**Moran test**

<table>
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<tr>
<th>Matrix</th>
<th>$I$</th>
<th>p-value</th>
<th>$I$</th>
<th>p-value</th>
<th>$I$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$</td>
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<td>0.0000</td>
<td>0.3649</td>
<td>0.0000</td>
<td>0.4220</td>
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<tr>
<td>$D$</td>
<td>0.4953</td>
<td>0.0000</td>
<td>0.4066</td>
<td>0.0019</td>
<td>0.1096</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
nomic distance matrix – $D$). The Moran’s $I$ statistics for variable $Y$ are very similar using both types of the connection matrix (0.4714 and 0.4953, respectively) with the proviso that units with a similar unemployment rate in 2019 showed slightly higher neighbourhood dependence. In contrast, the Moran’s $I$ coefficient evaluated considering the distance matrix was relevantly lower than in the case of the first-order contiguity matrix $W$) for two remaining variables. It means that the dependence of GDP per capita and educational attainment between neighbouring regions in a geographical space was stronger than between regions neighbouring in economic terms. Nonetheless, all determined Moran’s coefficients turned out statistically significant. This situation indicates the necessity of spatial dependence inclusion in the analysis of the relationship between the unemployment rate and economic growth, and educational attainment.

**Okun’s relationship models**

In the first part of the relationship analysis, the TSCS (pooled time series and cross-sectional) model was considered. *Table 2* presents the results of the estimation and verification of the pooled spatio-temporal model. This is Okun’s model extended with the educational factor.

The negative estimate of statistically significant parameter $\beta_1$ denotes that an increase in the GDP per capita causes an average decrease in the unemployment rate. This confirms the observations made by Arthur M. Okun. The value of the elasticity parameter indicates that an increase in the GDP by 1 percent provides to decrease in the unemployment rate averagely by 0.41 percent *ceteris paribus*. A higher strength of the influence shows the educational level, where an increase of 1 percent causes the average decrease in the unemployment rate averagely by 0.73 percent.

Moran test results indicate the presence of global spatio-temporal autocorrelation in the model residuals. Therefore, the significance of dependence between neighbouring regions in the light of both neighbourhood matrices was concluded. So the spatio-temporal models estimated considering geographical and economic neighbourhoods should be analysed.

In order to determine the character of spatio-temporal dependence, the Lagrange Multiplier tests were conducted. Tests statistics in the basic version ($LM_{err}$ and $LM_{lag}$) do not solve the problem of the model choice – both are statistically significant. Analysing $LM$ statistics in the robust versions of tests, we can conclude that the model with spatial factor in the error term for the $W$ matrix is better. In turn, for the economic distance matrix (marked as $D$), the model with a spatial lag of the dependent variable should be

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Standard error</th>
<th>t statistics</th>
<th>p-value</th>
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<td>$\beta_1$</td>
<td>-0.4194</td>
<td>0.0320</td>
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<tr>
<td>$\beta_2$</td>
<td>-0.7301</td>
<td>0.0925</td>
<td>-7.8960</td>
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<tr>
<td>$R^2$</td>
<td></td>
<td></td>
<td>0.1353</td>
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<table>
<thead>
<tr>
<th>Moran test</th>
<th>W</th>
<th>D</th>
</tr>
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<tbody>
<tr>
<td>$I$</td>
<td>0.4363</td>
<td>0.4257</td>
</tr>
<tr>
<td>p-value</td>
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<td>0.0000</td>
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</table>

<table>
<thead>
<tr>
<th>LM tests</th>
<th>Estimate</th>
<th>p-value</th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LM_{err}$</td>
<td>611.5708</td>
<td>0.0000</td>
<td>699.1660</td>
<td>0.0000</td>
</tr>
<tr>
<td>$LM_{lag}$</td>
<td>575.7326</td>
<td>0.0000</td>
<td>859.9580</td>
<td>0.0000</td>
</tr>
<tr>
<td>$RLM_{err}$</td>
<td>38.3855</td>
<td>0.0000</td>
<td>21.3310</td>
<td>0.0000</td>
</tr>
<tr>
<td>$RLM_{lag}$</td>
<td>2.5453</td>
<td>0.1106</td>
<td>182.1230</td>
<td>0.0000</td>
</tr>
</tbody>
</table>
chosen. Regardless of the results of the robust LM tests, both types of spatio-temporal models enriched with the spatial lags of explanatory variables were estimated.

Table 3 presents the results of estimation and verification of the spatio-temporal Durbin models (STDM) and spatio-temporal hybrid models (STHM), considering both connection matrices.

Similar to the TSCS model, estimates of statistically significant parameters $\beta_1$ and $\beta_2$ are negative, which confirms the positive influence of the GDP per capita and educational attainment increase on the labour market conditions in the EU regions. Nonetheless, the strength of impact is lower. The highest difference is observed in the estimate of the parameter $\beta_2$ in the models for the $D$ matrix—the 1 percent increase in educational attainment provides the average decrease in the unemployment rate around 0.4 percent (less than for the TSCS model around 0.3%). In all models, parameters $\rho$ and $\lambda$ are statistically significant, which confirms the necessity of including spatial factors in the analysis. Values of estimates of the spatial parameters within one connection matrix are very similar. Moreover, the estimates of both considered parameters are slightly lower in the case of models for the economic distance matrix. This indicates that regions neighbouring in the geographical space were more similar in the unemployment rate than the neighbours determined by the economic terms.

It is worth noting the statistical significance of the $\theta_1$ parameter in all estimated models. Except for the spatial Durbin model for the $W$ matrix (SDM_W), the estimate of this parameter is negative. Considering the geographical neighborhood, changes in the unemployment rate in the neighboring regions had a different impact than changes in the random processes or processes omitted in the model. In turn, shocks like an increase in the unemployment rate level or in the random processes in the neighbouring (from the economic point of view) units caused a significant decrease in the unemployment rate in a certain region. However, shocks in the random processes or omitted explanatory variables were slightly stronger.

The desirable characteristic of the models with the $W$ matrix is the lack of spatial autocorrelation in the models’ residuals. In the light of the Akaike Criterion (AIC) and the logarithm of likelihood values (Log-lik)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model</th>
<th>STDM_W</th>
<th>STHM_W</th>
<th>STDM_D</th>
<th>STHM_D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td></td>
<td>-0.3094</td>
<td>-0.3289</td>
<td>-0.3111</td>
<td>-0.3296</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0000)</td>
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the best model is the spatial Durbin model estimated using connection matrix $D$ based on the economic distance neighbourhood. Therefore, we can conclude about higher cognitive values of the models with the economic neighbourhood (regardless of the spatial autocorrelation presence in the model residuals). Statistical significance of the parameters $\rho, \beta_1$ and $\theta_1$ allows for quantifying the short-term spatial spillovers. Nevertheless, this issue is not the subject of the study. The quantification of the spatial effects is one of the directions of further analysis.

**Discussion**

The analysis presented in this paper shows a large variation in the unemployment rate between EU regions, which is a significant economic problem. The problem particularly refers to the southern and eastern parts of the European Union. The causes of this situation have a different character. The economic growth and education level of society are considered as the most important indicators influencing the labour market level. Estimated models confirm that the increase in Gross Domestic Product per capita and educational attainment (measured by the percentage of graduates in upper secondary and post-secondary school) significantly causes the decrease in the unemployment rate. Moreover, the other individual characteristics of regions influence the labour market situation. One of them is specific economic structure, i.e., if the considered regions are rural or industrial. A verification of Okun’s relationship for EU regions taking into account their specificity will be the subject of further research. Boďa, M. and Považanová, M. (2020) point out the necessity of diversifying regions by their specific characters estimating Okun’s relationship. In turn, Bonaventura, L. et al. (2018) verified this relationship in two gender groups in Italy. They inferred differences in the sensitivity level of the unemployment rate on changes in GDP per capita between males and females depending on the geographic location of the region.

Some of the authors so far analysed Okun’s relationship on the regional level. For example, Duran, H.E. (2022) showed its significance for Turkish provinces. He did not include additional explanatory variables apart from the economic growth level, the increase of which causes the decrease in the unemployment rate in all provinces. In turn, Melguizo, C. (2017) considered the connection between economic growth and the unemployment rate in Spanish provinces. She inferred the same type of relationship (with different strengths) throughout the area. Also negative sign of Okun’s coefficient for all regions of Slovenia obtained Dajcman, S. (2018). Palombi, S. et al. (2017) showed the same for Great Britain, analysing data for regions at a NUTS-3 level. Their study is one of few using spatial econometric models as a research tool for the verification of Okun’s relationship. Other analyses based on the spatial econometric approach were conducted by Montero-Kuscevic, C.M. (2011), Perreira, R.M. (2013), and Maza, A. (2022). In this research, additionally, the educational attainment level was included in Okun’s model, which is not found in many other studies.

The methodological approach used in this research differs from other approaches. Firstly, previous researches including spatial connections between territorial units based on the First Differences Method of Okun’s law. This analysis used the Fitted Trend and Elasticity Method enriched with the spatial trend. It is a new approach to establishing long-term dependencies in the formation of key indicators used in the investigation, which treats the trend wider than yet. In turn, the definition of one of the spatial connection matrices is new. It is an economic distance matrix built on the unemployment rate similarity between regions. As we saw in the results, changes in GDP per capita level in regions with similar unemployment rates influence stronger on the labour market conditions in the specific region, than changes in the regions directly adjacent. Spatial models in previous studies were estimated using neighbourhood matrices built based on the common land border or the geographic distance criteria. The weakness of this research is not consider-
ing the specific characteristics of regions, for example, a population composition, an economic structure, and other important indicators, such as an innovation level. These aspects will be the subject of further research.

Conclusions

The regional approach to the verify of Okun’s relationship has become more and more popular in macroeconomic analyses. Regardless of the regional disparities in the unemployment rate and the economic growth between NUTS-2 level units, the general dependence among these processes was confirmed in the European Union. The Okun’s elasticity parameter ($\beta_1$) in the estimated spatial models took a value similar to this, considered to be a benchmark around -0.3 (value received by Arthur M. Okun in his study). Moreover, the educational attainment turned out to be significant, and an increase in the percentage of graduates in upper secondary and post-secondary schools caused a decrease in the unemployment rate. In this connection, the first research hypothesis of the study was confirmed.

The economic similarity included in the models in the form of a neighbourhood matrix turned out to be statistically significant, so the similarity between regions related to the unemployment rate is relevant in Okun’s relationship verification. So this is the second type of connection, next to the repeatedly confirmed significance of the geographical closeness (in this study, too), which allows for an understanding of the formation of the relationship between unemployment and output growth. A comparison of the estimated spatio-temporal models shows that models with the economic neighbourhood (regardless of the certain imperfections) better explain the mentioned dependence, which confirms the second research hypothesis. It means that the regions similar in the unemployment rate levels are connected stronger in case of the relationship between economic growth and labour market conditions than the regions directly adjacent to each other.

The proximity in the sense of the similarity of the unemployment rate can explain the imitation effect related to regularities and rules introduced by the governments of regions. The patterning of the regional rulers’ behaviours from other provinces in the case of the labour market situation can provide similar changes in the labour market in a certain unit. It is also worth noting the policy of combating unemployment should be fitted to the regional specificity of the local labour market.

It is worth noting that in the adopted time range (2013–2019), all crises are omitted: (1) financial crisis in 2007–2009, (2) economic slowdown in 2012, and (3) COVID-19 pandemic from 2020 (due to lack of the data). In this connection, the relationship between the unemployment rate and output growth may be accepted as relatively stable in the European Union regions (which does not mean that not differentiated between units). In further research, this is worth concerning also the analysis of the regimes of the regions divided by the economic growth level and the impact of the COVID-19 pandemic on the mentioned relationship. This research will be enriched with the spatial effects quantified based on the estimated models and the use of other spatial connections matrices.

REFERENCES


