Contribution to regional division of Slovakia based on the application of the Reilly's model

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Abstract

The objective of the present article is an application of three versions of the Reilly's model in a regional division of Slovakia at two levels: from the point of view of potential natural gravity towards regional centres, and from the aspect of an alternative proposal of administrative division of Slovakia at the regional level. In this contribution the geometric version of the Reilly's model serves only as a complementary tool for an assessment of a regional organisation of Slovakia. A crucial part lies in the application of the topographic version; the oscillatory version possesses a correcting and refining role. Functional urban regions have been used as basic spatial zones in our analysis. The geometric version of the model is used for a preliminary assessment of possible influences of centres. Let alone several exceptions caused by physical geographical conditions (e.g. an overrated sphere of influence of Žilina) this version provides a relatively realistic image of the Slovak regional system.

Keywords: regional division, potential spatial interactions, Reilly's model, Slovakia

Introduction

Geographical space is not a homogeneous entity and individual geographical components are not distributed uniformly in the space, i.e. they are represented by different intensities in distinct regions. In most cases there is a natural tendency to balance these differences. As an example from physical geography the occurrence of horizontal flows of various types can be put forward. For instance in case of different values of atmospheric pressure the "polarity" is balanced by the flow of air masses. Such flows posses a character of vector or gradient.

An analogical situation can be witnessed also in case of human geographical partial components. Horizontal flows (of people, products, informa-

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tion etc.) occur in the social and economic environment as well. Geography mostly refers to them as the spatial interactions. They represent an aggregation of individual mobility and contacts. As for the flows of people they are conditioned by the activities of individuals. The spatial behaviour of individuals is affected by their needs and efforts to optimise spatial movements (or spatial location) in order to gain economic and social benefits. Socio-economic spatial interactions substantially affect the geographical organisation of the society and express the interdependence among the sections of geographical space (regions) of different hierarchical levels.

The primary information on spatial mobility of the population and spatial interactions is based on the migration data, particularly on the labour and school commuting. The labour commuting, which is the basic platform for regionalisation tasks, was recorded in the former Czechoslovakia for the first time in the 1961 census. Since then the data on main commuting flows have been available only in ten years intervals. Other data on the spatial interactions (for instance on passenger traffic volumes, attendance of shopping malls etc.) are very complicated to acquire and in some cases they are subject to business secret.

If such insufficiencies cannot be supplied by questionnaire surveys (these would be very demanding and practically not applicable on the whole state territory), geographers frequently resort to the use of potential spatial interactions when handling the regionalisation tasks. These approaches rely on the assumed non-homogeneity of geographical space and have been inspired by the laws of physics, for instance by the Newton law of universal gravitation.

The partial objective of the article is to apply basic geometric and topographic versions of the Reilly´s model on the territory of Slovakia. Spheres of influences of individual settlements reached by the geometric version will serve as a base for the selection of the centres and subsequent application of the topographic version of the Reilly´s model with a modified parameter. The primary objective of the article is, then, a delineation of regions in Slovakia aimed at putting forward an alternative proposal of an administrative division of Slovakia at the level of regions (i.e. NUTS 3), and their grouping into the NUTS 2 regions. This analysis will employ the potential spatial interactions. The proposal will attempt to take into account the rule of spatial justice and the size of the NUTS 2 regions recommended by the EU.

The delineation of the NUTS 3 regions in Slovakia is not correct and is subject to persistent harsh criticisms from the independent scientific public (for instance deliberate distribution of the Hungarian minority between the Bratislava, Trnava and Nitra regions, inadequate demarcation of the Spiš historical territory etc.). Our alternative proposal has made use of the Reilly´s model since the higher levels of the administrative system (NUTS 2 and NUTS

3 regions) cannot be delineated according to real commuting or migration flows (for instance the gravity zone of Bratislava reaches far eastern Slovakia). This claim is supported by the fact that such a task has not been undertaken by the Slovak geography so far. One of a few options of such a construction based on the quantitative data processing is presented in the article.

Theoretical background and methods

Geography (or spatial science) saw the first application of simple models inspired by the Newton law of gravitation in the 19th century (RAVENSTEIN, E.G. 1885). The issue of spatial interaction modelling had further developed during the inter-war period when William J. REILLY (1929, 1931) defined the so called law of retail gravitation, which was based on the real interactions observed in Texas during the second half of the 1920s. However, the theoretical explanation of socio-economic spatial interactions based on the gravitation concept occurred after the Second World War, when STEWART, J.Q. (1948) formulated his concept of social physics and used the term demographic force as an analogy to gravitation force used in the natural sciences. STEWART´s work reflected the conclusions reached by ZIPF, G.K. (1947) regarding the principle of minimum effort which is very important for the spatial interaction modelling since it is closely related to a "resistance" exerted by the geographical space towards the spatial interactions.

The law of retail gravitation has been further modified on the basis of the above mentioned works by CONVERSE, P.D. (1949) and HUFF, D.L. (1964). Converse mathematically expressed the breaking point between the spheres of influences of neighbouring centres, whereas Huff was the first to express the theoretical probability of choice of shopping centre by customers (not applied on the concrete territory). Derivation of the interaction models has not been limited to a mere analogy to the Newtonian physics. WILSON´s derivation is based on the entropy maximising method inspired by the second law of thermodynamics (WILSON, A.G. 1970) and it shows that the Reilly´s model and equation identifying the breaking point is only a special case of the so called unconstrained interaction model.

Originally the Reilly´s model was constructed as a tool identifying the retail attraction and was based on purely formal relations. It was applied mainly to determine the tendencies of the population to travel to selected centres in order to reach different types of services and to identify the borders of influence between centres within simple graphical schemes of the settlement system (e.g. FOTHERINGHAM, A.S. and O´KELLY, M.E. 1989). A different type of task was and mainly currently is a delineation of tributary areas of shopping centres, i.e. points carrying the masses are not conceived as settlements (e.g.

Lee, M.L. and Pace, R.K. 2005; Baray, J. and Cliquet, G. 2007). A note should be made here that not all mentioned works are products of geography but of spatial economy as well. Lee and Pace (2005) deal with a spatial distribution of retail sales between the shopping centres in relation to their mutual location in Houston, Baray and Cliquet (2007) besides the application of the gravity models discuss the possibility of mathematical morphological analysis for delineation of shopping centres' tributary areas.

In Slovakia the Reilly´s model was used by Očovský, Š. (1973) to delineate the tributary areas of shopping centres in Slovakia, in the Czech literature the topic was discussed by e.g. Maryáš, J. (1983), Hlavička, V. (1992) or Řehák, S. (2004). Although originally the Reilly´s model was intended to identify the retail attraction, currently it can also be used – despite some objections made by Berry, B.J.L. (1967), regarding however rather its original use – for the assessment of geographical organisation of a territory, suitability of its administrative division, of its historical or future development, and for general regionalisation tasks (Hubáčková, V. and Krejčí, T. 2007; Halás, M. and Klapka, P. 2010; Klapka, P. and Niedźwiedźová, K. 2010).

The law of retail gravitation formulated by Reilly, W.J. (1929) states that the portion of realised shopping visits in two competing centres (settlements) depends on the size of these centres (increasing size of a centre brings increasing portion of visits in it) and on the distance between these centres (increasing distance from a centre brings decreasing portion of visits in it). Mathematical expression of this relation is:

$$B_A / B_B = \left(M_A / M_B\right)^N \left(d_B / d_A\right)^n ,$$

where B_A, B_B are number of visits in centres A, B from the place (settlement) examined; M_A, M_B are masses (weights) of the centres A, B given by their population; d_A, d_B are distances from centres A, B. Parameter N was set by Reilly to 1 and parameter n to 2, which is of course the full analogy to the gravitation law.

Border of the influence spheres of two competing centres A, B is made by a set of points possessing the equal number of visits to the centres, i.e. $B_A/B_B = 1$. If we keep N = 1 we get by adjustments the equation

$$k = \sqrt[n]{\frac{M_A}{M_B}} = \frac{d_{AB} - d_B}{d_B}$$

valid in case that $M_A \geq M_B$ where d_{AB} is a distance of centres A, B; d_B is a distance of smaller of the two centres from the influence breaking point of these centres along their shortest link. In practice it means that the border between

influences of two centres is a set of points whose distance from the centre *A* is a *k*-multiple of the distance from the centre *B*. In case that $M_A \neq M_B$ we get a circle as a dividing set of points. Its construction is described in detail by Řehák, S., Halás, M. and Klapka, P. (2009).

Weights of centres M_A, M_B can be defined in various ways according to the character of phenomena to be modelled. In original works the weights were given either by the populations of centres or by the financial expression of the retail sales. In case of a delineation of the economic influence the econometric indices can be used, such as the size of shopping area, number of occupied job positions or the number of entrepreneurial subjects (used e.g. by Hubáčková and Krejčí, 2007). The results are basically analogous to those reached by the use of the population as the weight of a centre (see conclusions of several reports cited by Maryáš, 1983).

Löffler, G. (1998) in his complex outline states that generally the population, number of firms in the service sector, number of job opportunities and the so called "commuting balance" can be used as weights. However, it is the population that is the simplest and most universal factor expressing the weight of a centre. It is most suitable for general tasks of approximation of complex human geographical regionalisation of a territory and for proposals of administrative division of a territory as well (other indices can be distorted by a functional specialisation of some centres). In the present applications a potential (theoretical) attraction to a centre at a general level will be applied, therefore the most complex and universal factor, which is the population of the centres as of January 1, 2009, will be used. According to the way of areal delineation three basic versions of the Reilly´s model have been defined: geometric, topographic and oscillatory (Řehák, S., Halás, M. and Klapka, P. 2009), each of them having its reason in a particular orientation and phase of the research.

The simplest *geometric version* of the Reilly´s model works in isomorphic space, i.e. with distances in air kilometres, without the communication network being taken into account. The border of the influence spheres of two centres is always a circle, in the case of centres of equal weight this border is straight line. The advantage of the geometric version is seen particularly in cases of preliminary assessment of possible influences of centres when surveying larger territories, with a well designed communication network and without distinct natural barriers. This version can also be very well used for the identification of the influence spheres crossing the national border and in historically conceived tasks for generalised retrospective analyses of the settlement system.

The *topographic version* of the model does not work with an isotropic plain but with some more or less tangible geographical characteristics of a territory, for instance with communication network, which also reflects the

physical geographical conditions of a surveyed territory to a certain extent. This version already employs the spatial zones (e.g. municipalities) and with distances separating the centres of these spatial zones along the elements of the communication network, such as roads or railways. The border between the influence spheres is confined to the borders of the spatial zones, when each zone can be unequivocally assigned. The topographic version can be used for classic regionalisation tasks and for testing the suitability of the spatial division of a territory.

The *oscillatory version* of the Reilly´s model is not inherently aimed at the regionalisation but at the identification of transitory belts. The construction of these areas has its reason particularly when employing the spatial zones as in the preceding version. It identifies those zones that are located near the border of the influence spheres of the centres. Delineation of this transitory belt can be achieved by setting the span of the belt, for instance in a form $(0.9 \cdot k; 1/0.9 \cdot k)$. The oscillatory version of the model can be applied at the beginning of detailed study of attraction, but also at the final stages of research as a correction of the resulting regionalisation.

A special attention should be paid to the n parameter choice in the basic equation of the Reilly´s model. Maryáš (1983) reminds a long time discussion about the values of this parameter (e.g. also Schwartz, G. 1962) and claims that for centres of lower orders the needed tributary area had been reached by application of the distance parameter with the value at least three. The construction of the model shows that it can be calibrated by the n parameter estimation, while the classic version of the Reilly´s model had worked with the value $n = 2$. Reilly had chosen two having said that the mode of his sample had belonged to the interval from 1,51 to 2,50. If we strive to work exactly and take into account the attraction/gravity as the analogy to the laws of physics we have to use the value two. However, the parameter choice has to reflect the character of a phenomenon to be approximated or the nature of the application task (if we focus on the practical use).

The selection of centres is an important part of the Reilly´s model application. It can follow several criteria. The simplest is selection according to the size criterion where the population number of the centre or potential population of its influence sphere can be taken into account. The selection of the centres can use existing versions of regionalization based on the real interactions when it is possible (at a given hierarchical level) to adopt these centres.

In this article the geometric version of the model will serve only as a supplementary tool of the assessment of the regional division of the Slovak territory. Observed influence spheres of individual centres will be taken into account when selecting later the centres for individual variants of the possible territorial divisions. The application of the topographic version will bear the

major significance (see below), the oscillatory version will serve a practical purpose as a correcting and refining tool for the results of the topographic version. Since the Reilly´s model will be used for regionalisation with several different objectives, details of the method will be provided in the ensuing text in the places where it is applied.

Existing regionalisations of Slovakia, basic spatial units

The literature presents several regionalisations of Slovakia. One of the most important of them is the delineation of the so called daily urban systems, generally known as the functional urban regions (FUR further on). Bezák, A. (2000) has delineated two versions of functional urban regions (FUR 91-A and FUR 91-B) on the basis of 1991 labour commuting data having employed a relatively sophisticated method. The second system (marked as FUR 91-A), except for the inner coherence and outer separation, has met the criterion of minimal population size of the region (35,000 inhabitants) as well.

In this article (precisely when applying the topographic version of the Reilly´s model) Bezák´s FURs 91-B have been used as basic spatial units. They had been delineated only on the basis of a scientific data processing without inclusion of a minimum size of a region, and at the same time considered to present the primary regionalisation of the Slovak territory convenient for the analyses of the spatial data or for further geographical processing. Slovak districts cannot be used as basic regions, since their delineation was purposefully and strongly political and the criterion of spatial justice was not obeyed. In the north of Slovakia the status of a district capital was assigned even to the small centres (such as Bytča, Kysucké Nové Mesto, Turčanské Teplice) and these districts do not match their counterparts in the southern Slovakia in terms of their size (for instance Nové Zámky, Levice, Rimavská Sobota, Trebišov).

In geographical scientific literature no regionalisations of Slovakia have been found at a hierarchical level higher than the system of FUR, that would aim directly at the proposals of the territorial administrative division. It can be considered as a relatively serious deficiency, since particularly this level (NUTS 3) acts as regional self-governments that are delineated in a very poor and biased way in Slovakia (their delineation in 1995–1996 was strongly influenced by the political intentions of the government coalition of the time). Hierarchically higher regional system delineates NUTS 2 regions, i.e. basic territorial units of the EU countries serving (among others) for the distribution of main financial flows into the regional policy of the EU. We register only several scientific proposals of the division to the regions, i.e. NUTS 3 level (e.g. Bačík, V. and Sloboda, D. 2005; Sloboda, D. 2006), which are all already better than current territorial administrative division, but are not supported

by the exact quantitative processing of the data on real or potential population flows. Moreover, these proposals employ the districts delineated in 1996 as basic spatial units, which had not been constructed very transparently either (see the preceding paragraph).

Application of the geometric version of the Reilly´s model on the Slovak territory

The geometric version of the model serves in the article only illustrative purposes and presents a preliminary view of the possible application of the Reilly´s model. As the parameter we have set the most frequently used value $n = 2$ and the selection of centres followed a simple population criterion. The original intention for the centre selection has been the level of 25,000 inhabitants, which has been later lowered to 24,000. The reason for this has been an inclusion of Rimavská Sobota (its population was 24,446 as of January 1, 2009), which could be in further applications, regarding its location in less exposed areas in terms of the population, seen as a potential regional capital.

In this chapter we turn our attention to a brief commentary of the geometric version results of the model *(Figure 1)* in relation to major geomorphologic barriers or to the comparison of the results of the topographic version of the Reilly´s model.

Despite the fact that the geometric version of the Reilly´s model cannot be completely ideally applied on the dissected territory in terms of landforms, it is generally able to approximate the settlement pattern and regional structure of Slovakia. Even when using the direct geometric distance it is possible
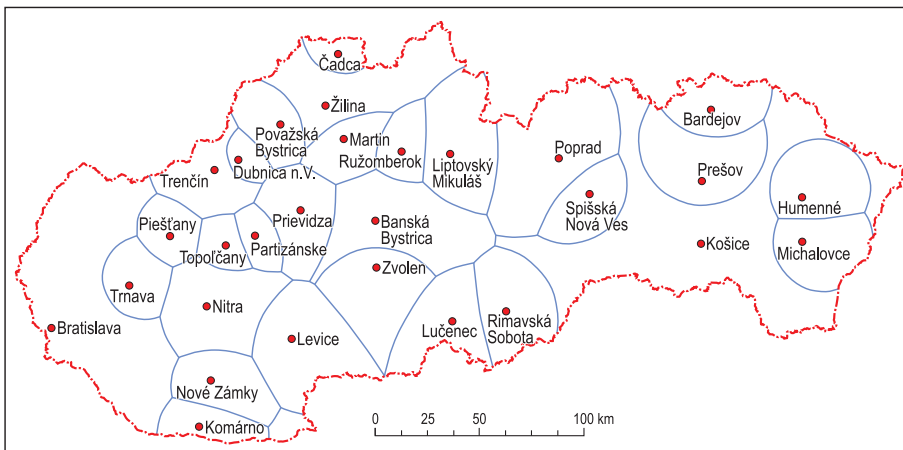

Fig. 1. Potential influence of centres of settlement system in Slovakia , 2009

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to identify the main direction of the so-called central Slovak communication barrier (more in Lukniš, M. 1985) mainly along the borders of influence spheres of Banská Bystrica or Prievidza (in the north-eastern part), Martin, Ružomberok and Liptovský Mikuláš. In comparison to the real situation the Prievidza influence sphere is actually reaching southwards (the influence of Žiar nad Hronom and Žarnovica, both having less than 24,000 inhabitants, has not been taken into account). The same situation holds true for the Poprad and Spišská Nová Ves influence spheres, when the influences of Rožňava and Revúca have not been considered.

The Bratislava influence sphere is relatively compact according to the geometric version, forming circa 50 km wide crescent reaching approximately 80 km to the north and 80 km to the southeast. On the contrary the Košice influence sphere forms several lobes or projections towards Rožňava, Stará Lubovňa, Medzilaborce, and Trebišov (all towns below 24,000 inhabitants). The spheres of influence of Bratislava and Košice have the so called exclaves, i.e. they do not obey the contiguity principle. The influence of Bratislava, intersected by the sphere of influence of Komárno, reaches the region of Štúrovo; the influence of Košice, intersected by the spheres of influence of Humenné and Michalovce, reaches the north-easternmost part of the country.

The geometric version overrates the influence sphere of Žilina and pushes it significantly to the north-east, where it comprises a large part of the Orava region. This territory would actually belong to Martin or Ružomberok (unless the influence of Dolný Kubín is taken into account), particularly as a result of very poor transport connection of the Kysuce region (and thus Žilina) to the Orava region.

In other instances the geometric version of the Reilly´s model relatively aptly approximates the basic features of the Slovak regional system (e.g. central Považie region etc.) and in general offers a relatively relevant information.

Application of the topographic version of the Reilly´s model on the Slovak territory: potential natural interactions

Attempting to approximate the potential natural interactions by the topographic version of the Reilly´s model, it is necessary to use the variant directly following the Newton´s law of gravitation, i.e. the variant with the value of the parameter $n = 2$. This fact has been tested on the territory of the Czech Republic (Halás, M. and Klapka, P. 2010), when the areas of the municipalities with extended authority (further on referred to as MEA areas) have been used as basic spatial units. The comparison of potential natural interactions reached by the topographic version of the Reilly´s model (using the parameter $n = 2$) to the real commuting interregional flows (i.e regionalisation by Hampl, M. 2004)

has provided a high degree of correlation. The majority of the MEA areas has been a part of the same mezzo-region; 94.8% of the population belonged to the same mezzo-region according to both methods. Moreover, out of remaining 5.2% of the population that has been assigned to different mezzo-regions, almost one half (exactly 2.5% of the population) is concerned with the trade-offs between the Hradec Králové and Pardubice mezzo-regions. It is caused by the advantageous location of the city of Pardubice on the major railway line, while this study takes into account the road distances only.

As it has been already mentioned, the basic spatial units (zones) that have been tested in terms of their affinity towards centres are FURs 91-B, when the road distances among their centres have entered the model as a distance variable between the zones. The model thus has used the road distances between our centres and FUR 91-B centres. The distances have been set by the Škoda Auto route planner (www.skoda-auto.com/cz) and they have had a character of the fastest connection in terms of the time needed. Each FUR 91-B has been assigned in the way described in the method above to the so-called mezzo-region (approximately the level of the NUTS 3 region). During the first phase 8 regional capitals have been defined as the mezzo-regional centres *(Figure 2)*, later four others have been supplemented *(Figure 3)*.

The main results are generally comparable to the geometric version of the model. The dominance of larger centres documents a regular co-existence of Bratislava with such centres as Trenčín, Trnava and Nitra in the west of Slovakia and similar situation occurs in the case of Košice and its co-existence with other centres in the east. Bratislava forms the influence sphere reaching along the borders with the Czech Republic, Austria and Hungary (from the town of Myjava to the town of Šahy). This sphere is significantly conditioned by the eccentric position of Bratislava within the state territory of Slovakia. The influence sphere of Košice comprises almost the whole eastern Slovakia with the exception of small tributary area of Prešov that is made up by five FURs only. The pattern in the central Slovakia (cities of Žilina, Banská Bystrica and partly also Trenčín and Nitra) approximates the regional administrative division quite well, since this territory lacks a significantly dominant centre *(Figure 2)*.

The cities of Trenčín and particularly of Trnava form reduced influence spheres in comparison to other regional capitals. Therefore other centres capable of forming the influence spheres comparable to those of Trnava and Trenčín have entered the second phase of the topographic version of the Reilly´s model (aimed at potential natural interactions). The cities of Martin, Poprad and Michalovce have not been doubted as regional centres in this respect. In the south of Slovakia a decision has had to be made whether insert the city of Lučenec or Rimavská Sobota in the set of the regional centres, since their spheres of influence have considerably overlapped. The final decision has favoured Lučenec, since it has been able to attract more population in this version.
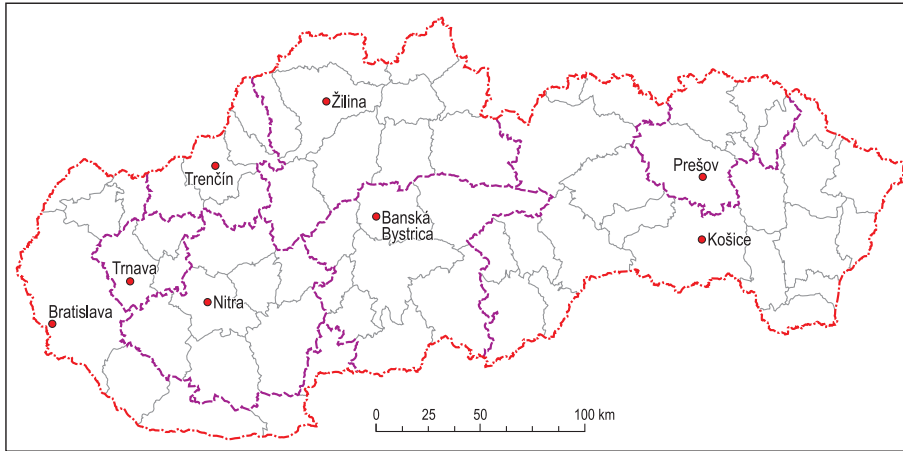
Fig. 2. Regional division of Slovakia based on potential natural interactions (8 centres)
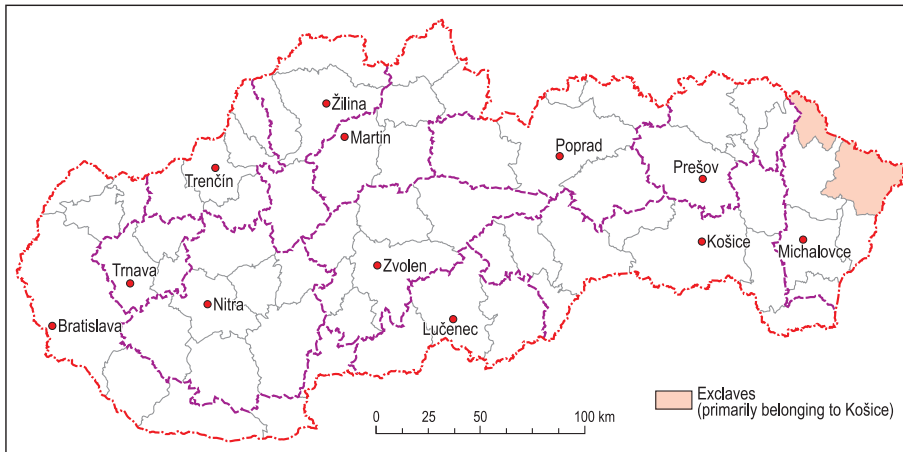

Fig. 3. Regional division of Slovakia based on potential natural interactions (12 centres)

When compared to the preceding situation the regional pattern in the west of Slovakia has not been adjusted and the central Slovak communication barrier has remained in its extent and position as well. The larger part of the influence sphere of Žilina has been attracted by the city of Martin (including the whole Orava region and the west of the Liptov region) and the FUR Liptovský Mikuláš has joined the regional centre of Poprad. The south of the influence sphere of Banská Bystrica has been taken over by the city of Lučenec, which has also attracted the FUR of Rimavská Sobota originally being a part

of the Košice region. In the eastern Slovakia the sphere of influence of Prešov has retained its extent. The part of the Spiš region, originally belonging to the regional centre of Košice, has been transferred to the regional centre of Poprad. Finally the Zemplín region has joined the sphere of influence of Michalovce.

Should the contiguity constraint of the delineated region be obeyed, the results in the north east of the state territory had to be corrected (the FURs Medzilaborce and Snina). These two exclaves are primarily attracted to the influence sphere of Košice according to the potential natural interactions (this fact is witnessed in the geometric version of the model as well – *fig. 1*). Their final regional affiliation has been determined by their secondary affinity, then. The population characteristics of the delineated regions are presented in *Table 1*.

Table 1. Population characteristics of regions of Slovakia (potential natural interactions)

Regional capital	Population in 1,000			Share of capital in population, %
	Total	Capital	Hinterland	
Bratislava	1,026.4	428.8	597.6	41.8
Nitra	738.5	84.1	654.4	11.4
Košice	667.9	233.7	434.2	35.0
Žilina	528.7	85.3	443.3	16.1
Poprad	408.6	54.6	354.0	13.4
Banská Bystrica	361.8	80.1	281.7	22.1
Prešov	344.2	91.3	253.0	26.5
Trenčín	333.3	56.8	276.5	17.0
Martin	298.0	58.4	239.6	19.6
Michalovce	256.3	39.5	216.7	15.4
Trnava	241.7	67.7	173.9	28.0
Lučenec	208.2	27.5	180.6	13.2
Total	*5,413.5*	*1,307.9*	*4,105.6*	*24.2*

Application of the topographic version of the Reilly´s model on the Slovak territory: proposals of territorial division

The attempt to propose the alternatives to the territorial administrative division (further on only territorial) of Slovakia by the application of the topographic version of the Reilly´s model has employed a higher value of the parameter *n*. The application of the higher parameter is necessary because when constructing a possible territorial administrative division, the rule of spatial justice has to be obeyed. It ensures that the extreme most locations of municipalities in individual regions have the comparable distance from their regional capitals. The value of the parameter has been set on the basis of extensive statistical testing.

In the preceding study on the Czech Republic (HALÁS, M. and KLAPKA, P. 2010), the F-test (with the level of significance 0,05 used for the comparison of the current territorial division and regionalisation according to the topographic version of the Reilly´s model and increasing the values of *n* gradually by 0,1) has produced the optimal value of the parameter *n* for the purpose of territorial administrative division (*n* = 5). Statistical sets in both compared divisions have been acquired as the maximum distances of the MEA area capital from the selected regional centres. The level of significance 0.05 has been reached also at the parameter with the value 5.0 (test criterion *F* has counted for 3.04 and for the last time it has been lower than the critical value of F-distribution for $m \times m$ degrees of freedom $F = 3.28$; the number of regional centres has been $m + 1$.

While F-test demands for a comparison with other (preferably existing) pattern, its application on the Slovak territory would not be correct. The current territorial administrative division of Slovakia, including the selection of eight regional capitals, can be considered as insufficient from the scientific point of view and thus cannot be used for statistical testing. Therefore, the value *n* = 5, statistically tested in the Czech Republic, has been applied in case of Slovakia as well. This approach appears to provide an optimal compromise between the elimination of increased influence of the largest centres (Bratislava and Košice) and the partial fulfilment of the principles of spatial justice. The argument for the use of the same parameter lies in the fact that both countries (Czech Republic and Slovakia) have a similar disproportion in hierarchy of their largest centres in terms of their order (seen for instance also in the Zipf curve etc.). These centres serve as potential capitals of NUTS 2 and NUTS 3 regions, then.

The selection of regional capitals (centres) is an important question. During the first phase the current regional capital, except for Trnava and Trenčín, have been employed. The two exceptions mentioned are not capable, due to their relative location to Žilina and particularly Bratislava, of forming the influence spheres comparable to the remaining six regional capitals. This variant has also substituted the city of Zvolen for Banská Bystrica. The reason for this step has been twofold: 1) Zvolen is capable of generation of larger tributary area, in comparison to the city of Banská Bystrica it attracts the FUR Rimavská Sobota in addition; 2) the variant with six resulting regions asks that Zvolen/Banská Bystrica should comprise the FURs in the Jihoslovenská kotlina basin and the city of Zvolen possesses a more advantageous central location within the resulting region, i.e. the sum of distances (simple and weighted by the population) from all centres of FURs to Zvolen is lower than to Banská Bystrica and thus the city of Zvolen appears to be a better option for the regional capital in terms of the principle of spatial efficiency.

The result *(Figure 4)* can be considered as optimal: six selected centres (regional capitals) manage to form comparable natural tributary areas and generally this variant of the possible territorial administrative divisions seems
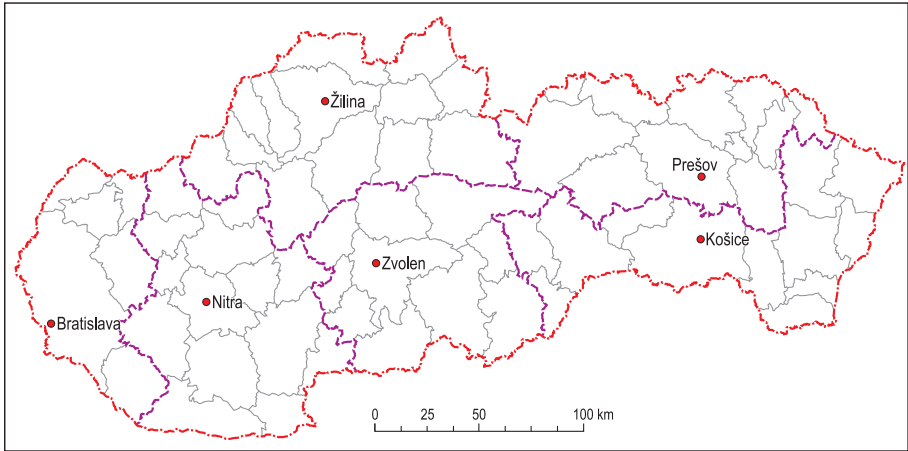
Fig. 4. Proposal of possible administrative division of Slovakia (regional level, 6 centres)

Table 2. Population characteristics of proposed regions of Slovakia (potential modified interactions, 6 centres)

Regional capital	Population in 1,000			Share of capital in population, %
	Total	Capital	Hinterland	
Žilina	1,129.0	85.3	1,043.7	7.6
Nitra	1,118.9	84.1	1,034.8	7.5
Bratislava	994.1	428.8	565.3	43.1
Košice	824.8	233.7	591.1	28.3
Prešov	763.4	91.3	672.1	12.0
Zvolen	583.3	42.5	540.8	7.3
Total	*5,413.5*	*965.7*	*4,447.8*	*17.8*

to be balanced. The population characteristics of the delineated regions are presented in *Table 2*.

Even though the territorial division of Slovakia with six regions can be seen as appropriate, it unfortunately does not match the EU criteria of the recommended population size of the NUTS 3 regions. It should vary according to the rules between 150,000–800,000 inhabitants. The optimal number of regions in Slovakia following the EU normatives is somewhere between 12 and 16. Therefore, the Reilly´s model for such a pattern of regional division has been applied once more. The possibility of merging the NUTS 3 regions into the NUTS 2 regions has been taken into account in the examination as well. The NUTS 2 regions should have the population between 800,000 and 3,000,000 according to the EU recommendations.

The first important step is a selection of centres, again. Eight regional capitals entered the model, the city of Banská Bystrica has not been substituted for by the city of Zvolen this time, since neither of the two arguments used in the variant with six regions has remained valid. Without any doubt the cities of Martin, Poprad, Michalovce and one of the pair Lučenec and Rimavská Sobota has had to be included in the set of the regional capitals. Using $n = 5$ the model has produced such a regional pattern that has had to be refined in several cases. First, the tributary area of Nitra has appeared to be too large and it exceeds together with the city of Nitra itself the level of 800,000 inhabitants. It seems to be necessary to form a region with a centre in Komárno or Nové Zámky, then. This adjustment makes the region of Nitra to fulfil the demanded size level and proves that the cities of Komárno/Nové Zámky are capable of forming the area comparable to other potential regions (i.e. exceeding the level of 250,000 inhabitants) as the only other centres in Slovakia. Finally the city of Nové Zámky seems to act as a more advantageous centre of the region due to its central location in the formed region and due to the fact that it meets better the demands of the spatial efficiency (the sum of distances weighted by the population from all municipalities is lower in case of Nové Zámky than in case of Komárno). No other city or town is able to form a tributary area comparable to the above-mentioned examples, therefore 13 regional capitals appear to be an optimal number taking into account the recommended size criteria.

All other transfers of the FURs from one region into another have been based on the application of the oscillatory version of the Reilly´s model only, which means in cases when the tested FUR has been located at the boundary of the influence spheres of two regional capitals within the span $(0.9 \cdot k; 1/0.9 \cdot k)$ (see methodology). Thus the region of Trnava has been joined by the FUR Skalica (otherwise the region of Trnava would have been too small and the region of Bratislava too large) and by the FUR Myjava. The FUR Liptovský Mikuláš has been transferred from the region of Poprad to the region of Martin, while preserving the historical border between the Liptov and Spiš regions, and, moreover, this transfer has enabled a simpler division to the hierarchical higher NUTS 2 regions. The choice of the regional centre between the cities of Lučenec and Rimavská Sobota has remained question that has not been easy to answer. The city of Lučenec manages to form a little larger area though (the difference is in the FUR Veľký Krtíš - the variant of the centre in Lučenec, and in the FUR Revúca – the variant of the centre in Rimavská Sobota), but the FUR Veľký Krtíš oscillates between Banská Bystrica and Rimavská Sobota, and the FUR Revúca has a poor accessibility to the proposed regional centre in Lučenec and never can be assigned to its region. There is just the only way to solve these problems satisfactory. If the city of Rimavská Sobota is made a regional capital than both questioned FURs Rimavská Sobota and Veľký Krtíš remain in the region. Other variants only raise other and more even problematic questions to cope with.

The resulting alternative proposal of the territorial administrative division with 13 regions is presented in *Figure 5*. It also shows an alternative division to the NUTS 2 regions: the current names West, Central and East can be preserved. All three proposed regions manage to follow the demanded population span, individual treatment of the city Bratislava is considered as irrelevant. Bratislava is not a metropolis with a million of inhabitants and in this phase it is not necessary to approach to the division of the NUTS 2 regions in an expedient manner in order to maximise the financial inflow from the EU funds. The population characteristics of the delineated regions are presented in the *Table 3*.
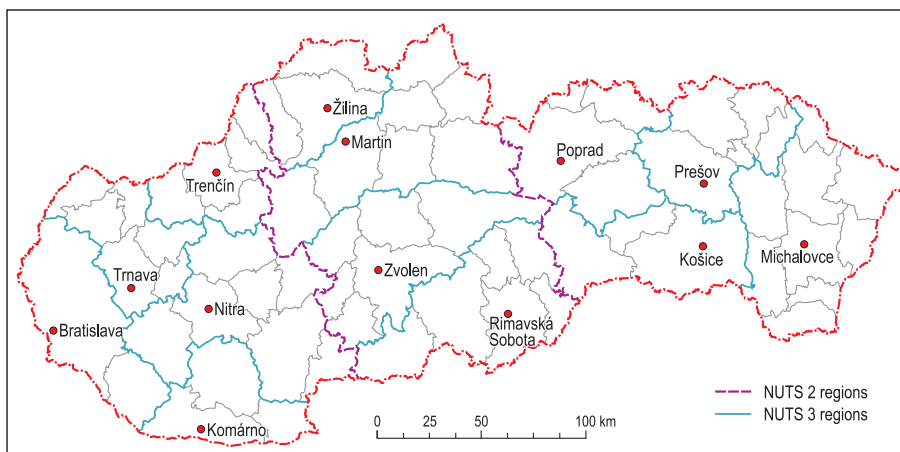

Fig. 5. Proposal of possible administrative division of Slovakia (regional level, 13 centres)

Tab. 3. Population characteristics of proposed regions Slovakia (potential modified interactions, 13 centres)

Regional capital	Population in 1,000			Share of capital in population, %
	Total	Capital	Hinterland	
Bratislava	735.6	428.8	306.8	58.3
Nitra	630.8	84.1	546.8	13.3
Martin	515.3	58.4	456.8	11.3
Michalovce	436.5	39.5	397.0	9.1
Košice	430.0	233.7	196.3	54.3
Žilina	386.8	85.3	301.5	22.1
Trnava	372.6	67.7	304.9	18.2
Banská Bystrica	361.8	80.1	281.7	22.1
Prešov	344.2	91.3	253.0	26.5
Trenčín	333.3	56.8	276.5	17.0
Poprad	333.2	54.6	278.6	16.4
Nové Zámky	267.5	40.5	227.0	15.1
Rimavská Sobota	265.8	24.2	241.5	9.1
Total	*5,413.5*	*1,340.5*	*4,073.0*	*24.8*

Conclusion

From the standpoint of natural conditions and prerequisites and from the standpoint of socio-economic regional disparities Slovakia is a considerably diversified country, therefore its regionalisation at any hierarchical level is a very demanding challenge. The results reached can be influenced to a considerable extent by the choice of regionalisation criteria, therefore these tasks cannot be taken as completed and could become a subject of further discussions.

The regionalisation employing the potential spatial interactions and the comparison of the results to the current territorial division of Slovakia (or to its proposals) has shown according to our opinion the viability of the Reilly´s model and its versions also in the solution of current regionalisation tasks. The results can be used also for the assessment of the characteristics of the settlement system in Slovakia and regional influence of its centres. The application of the Reilly´s model can vary according to the nature of the solved task.

The resulting regionalisations have confirmed several well-known facts. The specific position of the cities of Bratislava and Košice in the regional and settlement systems of Slovakia remains unchallenged. The capital city of Bratislava, despite its eccentric location, can be seen as a natural centre of the country, while the city of Košice is able to supply some functions of the country´s capital for the territory of the eastern Slovakia. Lukniš´ macro-regionalisation into two central and two corridor regions has been confirmed as well, while the central Slovak communication barrier can be identified in all results of the regionalisation tasks.

Other centres, excluding Bratislava and Košice, are classified at the minimum as one or two hierarchical levels lower. None of them significantly dominates since their development is limited either by natural barriers or the location within the vicinity of two largest centres. The absence of one dominating centre and significant geomorphologic barriers causes that the regionalisation of the central Slovakia is less problematic. Therefore the smallest number of the differences between individual proposals of this article and between current and past territorial divisions is witnessed in this territory.

The resulting regionalisations are significantly dependent on the selection of centres, while this issue is considered as a key question in the regionalisation tasks. The variant choices of centres can considerably affect the potential territorial divisions (e.g. Poprad, Michalovce etc.). In this place 12–13 years old discussion of the potential "Komárno region" can be mentioned when the main argument raised has been homogeneity instead of nodality. The alternative proposal of 13 regions put forward in this article comprises this independent region in the south of Slovakia (with the centre in the city of Nové Zámky). However, a note should be made here that the method presented in

this article is a mere theoretical expression of spatial interactions and is not able to reveal a possible influence of national differentiation of the territory on the existing nodal relations.

The results that have been reached in the article are in our opinion interesting in any case. They confirm the insufficient and problematic territorial delineation of current regional self-governments including the selection of their centres. A comparison of the results reached by the use of potential spatial interactions with the regionalisations constructed on the basis of real commuting or migration relations would be a very interesting research task for the future, then.

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Hungary in Maps

Edited by
Károly Kocsis and Ferenc Schweitzer

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'Hungary in Maps' is the latest volume in a series of atlases published by the Geographical Research Institute of the Hungarian Academy of Sciences. A unique publication, it combines the best features of the books and atlases that have been published in Hungary during the last decades. This work provides a clear, masterly and comprehensive overview of present-day Hungary by a distinguished team of contributors, presenting the results of research in the fields of geography, demography, economics, history, geophysics, geology, hydrology, meteorology, pedology and other earth sciences. The 172 lavish, full-colour maps and diagrams, along with 52 tables are complemented by clear, authoritative explanatory notes, revealing a fresh perspective on the anatomy of modern day Hungary. Although the emphasis is largely placed on contemporary Hungary, important sections are devoted to the historical development of the natural and human environment as well.

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This project serves as a preliminary study for the new, 3rd edition of the National Atlas of Hungary, that is to be co-ordinated by the Geographical Research Institute of the Hungarian Academy of Sciences.
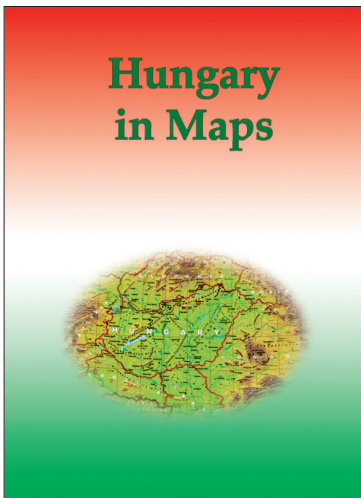
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